Fundamentals to motion

Three introductory concepts

Vectors

Scalars

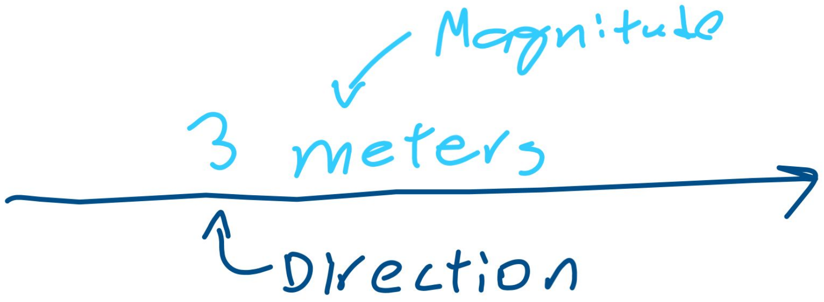
Scalars are numbers, and they have a single numerical value.

If I asked how many dogs do you own? You'd respond with a scalar like: 101

Vectors

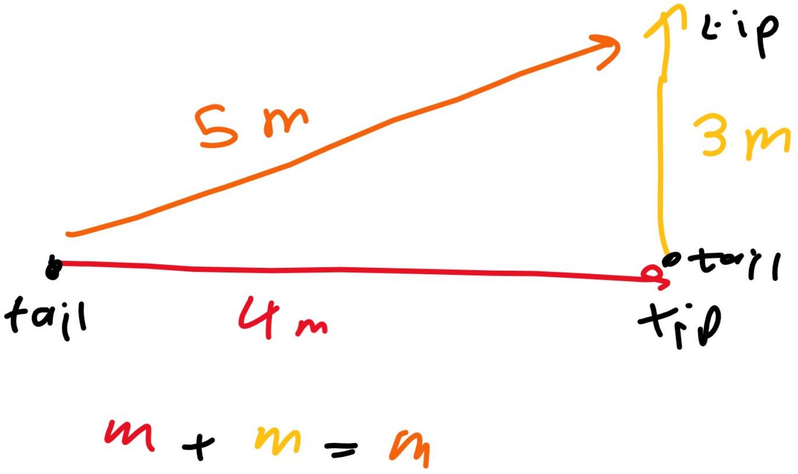
What if I asked you instead to tell me how to get to your school? You'd probably say like to 3 miles (a scalar), but you'd also tell me to go to the east.

Vectors have 2 parts:
Magnitude: how large it is (3 miles)
Direction: where (east)



Adding vectors

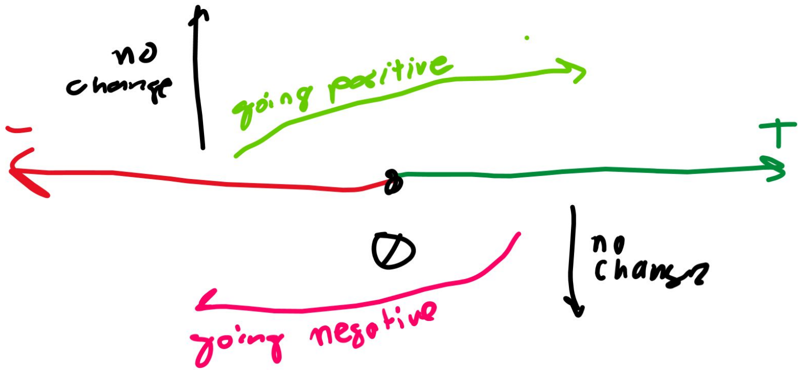
Tip to tail. The sum of two vectors is a new vector made by drawing from the tail of one to the tip of another.



Reference points and directions

These tell you whether you are going positive or negative

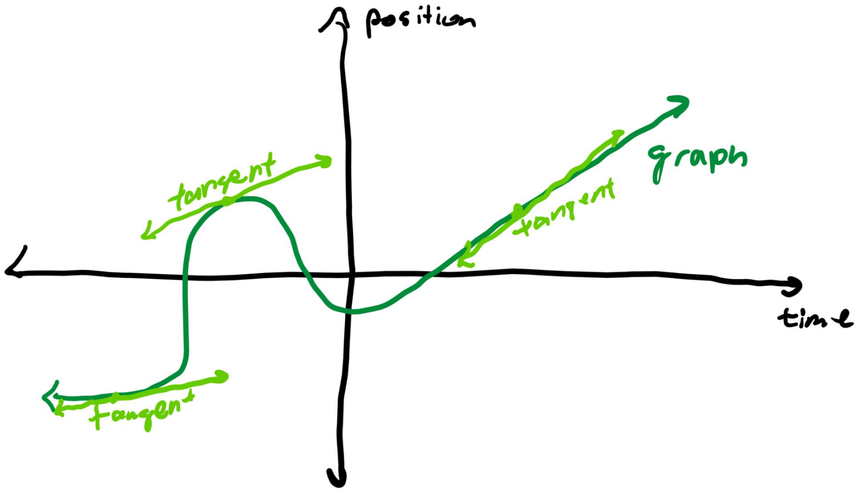
If we say going right is positive, we know going left is more negative, but going up and down does not change anything



Derivatives

What is a derivative?

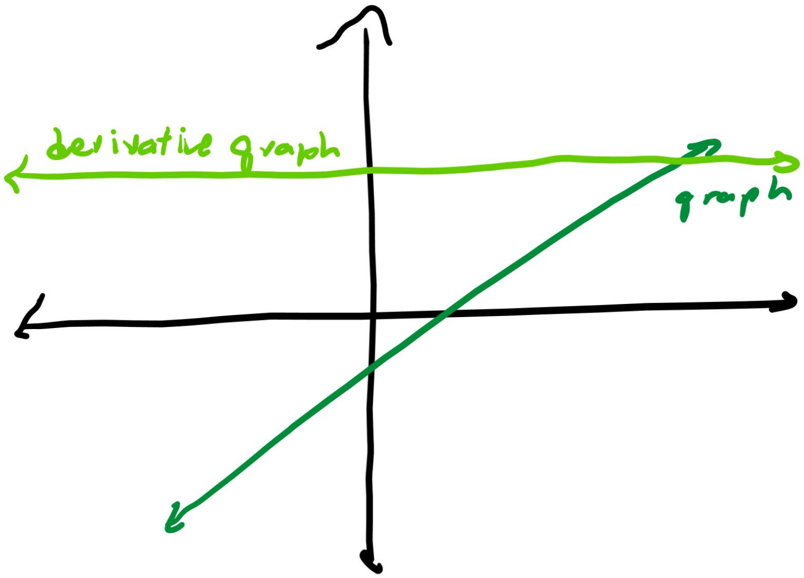
It’s a slope. On a graph, the derivative is the slope of the tangent line at a certain point.



Slope of a line

We know the slope of the line f(x) = 2x would be 2, but what does that mean?

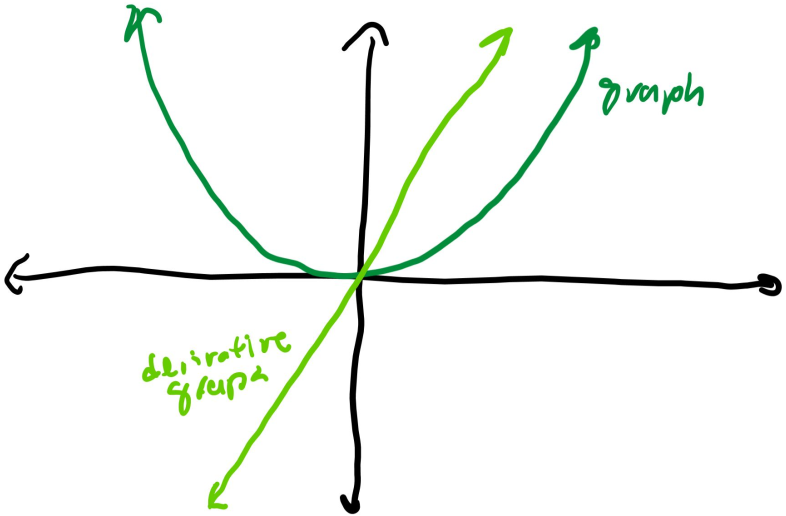
That means that the slope at every point on the line is 2, and we can graph the slope as y = 2



Slope of a not straight line

The slope on a curve like $y = x^2$ changes: first it's very negative, then it begins to increase until it's very positive, just like how $y = 2x$ starts very negative but becomes positive later

You can estimate the slope at a certain point by drawing a "tangent line" that touches the curve and calculating the slope of that line



Power rule

We can use this to find the exact equation for the derivative of a curve

The derivative of x^n is nx^{n-1}

The derivative of x^2 is 2x and the derivative of x^7 is 7x^6

another symbol for a derivative

$$\frac{d}{dx} x^n = (x^n)' = nx^{n-1}$$

a symbol for a derivative

Scalar multiplication and addition

If you have something like 6x^2, the derivative is 6 * 2x = 12x.

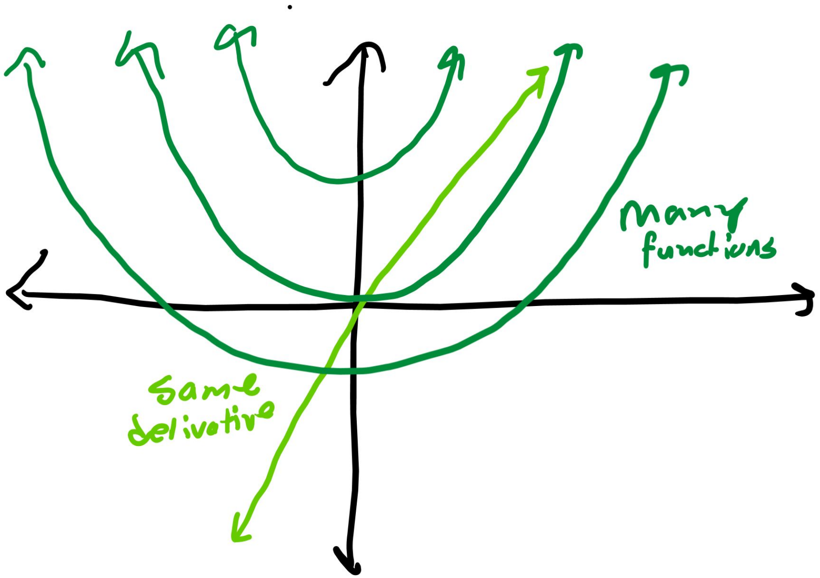
If you have x^3 + 7x^5, the derivative is 3x^2 + 7 * (5x^4) = 3x^2 + 35x^4

$$\frac{d}{dx} c\,f(x) = c\,f'(x)$$

$$\frac{d}{dx}\left(f(x) + g(x)\right) = f'(x) + g'(x)$$

Many functions with the same derivative

x^2, x^2 + 1, x^2 + 382, etc. all have the same derivative of 2x



Second derivatives (and more)

You can take a derivative more than once; if you do it twice, that's called taking the second derivative

The derivative of x^2 is 2x, so the seco derivative of x^2 is the derivative of 2x which is just 2

$$\left(\frac{d}{dx}\right)^2 f(x) = f''(x)$$

$$\left(\frac{d}{dx}\right)^3 x^3 = \left(\frac{d}{dx}\right)^2 3x^2 = \frac{d}{dx} 6x = 6$$

Antiderivatives

Is the antiderivative of 2x: x^2 or instead x^2 + 1?

We don't know, so we say it's x^2 + C where C stands for a constant

$x^{n}{}' = nx^{n-1}$ so the antiderivative of nx^{n-1} is x^n

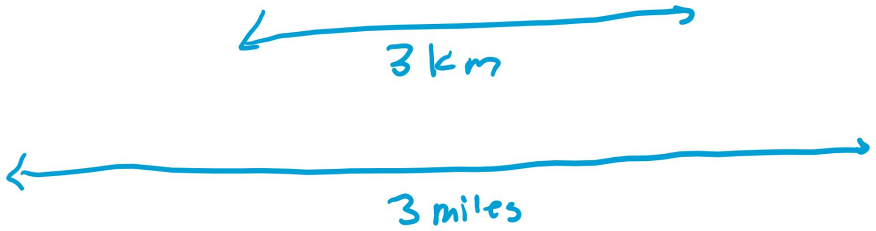
The antiderivative of x^n is $\frac{x^{n+1}}{n+1}$

Units

What are units

Units are a way we can measure things.

If you tell someone how far away something is, you don't just say 3. You say 3 kilometers or 3 miles. Those are very different.



Using units tactically

You can manipulate calculations to get certain units. For example, 6 meters divided by 2 seconds gives 3 meters per second.

If you know speed is measured in meters per second and you are given 12 meters and 4 seconds, what is the speed?

Kinematics

Position, velocity, acceleration

Position vs Distance

Position: a vector describing where something is from a certain reference point (a vector)

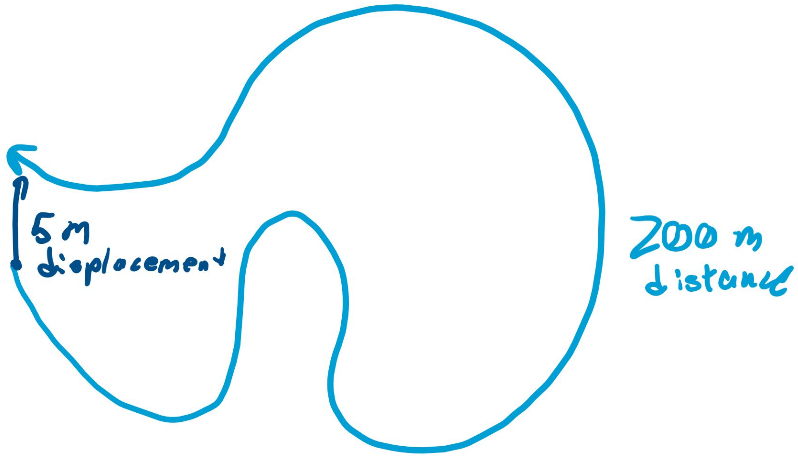
Distance: how far away something is from a certain reference point (a scalar)

Measured in meters (m)

Displacement

Distance: how far you traveled in total

Displacement: how far away you ended up from where you started

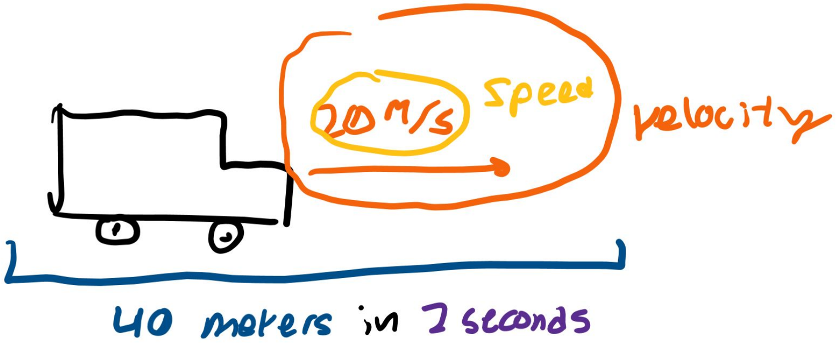


Velocity vs Speed

Velocity: a vector describing the change in position over time

Speed: a scalar describing the change in distance over time

Measured in meters divided by seconds, or meters per second (m/s)

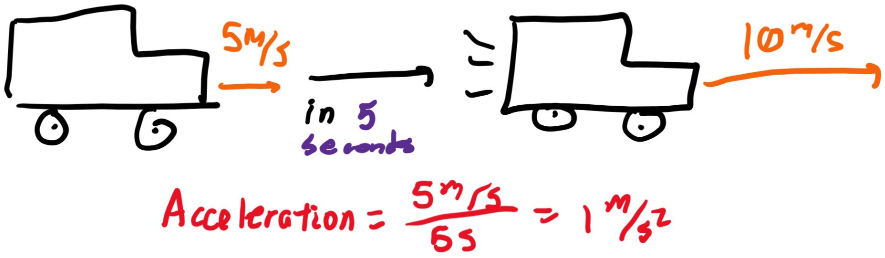


Acceleration

Change in velocity over time

How quickly your velocity is changing in a certain amount of time

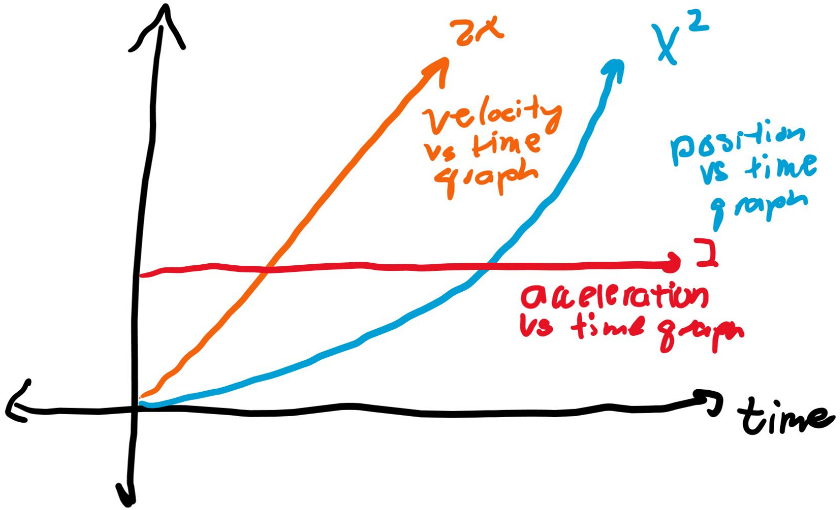
Measured in meters per second per second or meters per second squared (m/s/s = m/s^2)



Using derivatives

The slope of a position vs time graph would be speed. The slope of a velocity vs time graph would be acceleration.

If the equation of the position vs time graph is x(t) = t^2, then you know the velocity graph is 2t.



Using antiderivatives

You can go the other way as well. If you know acceleration is 4t^3, then you know velocity is t^4 + C.

If v(t) = t^4 + C, then when time is 0, we know v(0) = C, so we can instead write v(t) = t^4 + v_0 where v_0 is the initial velocity

Acceleration = $a(t) = 2\,m/s^2$

Velocity = $v(t) = v_0 + 2\,m/s^2 \cdot t$

Position = $x(t) = x_0 + v_0 t + 2\,m/s^2 \cdot \frac{t^2}{2}$

$= x_0 + v_0 t + 1\,m/s^2 \cdot t^2$